Estimation of Technical Inefficiency and TFP Growth via an Input Distance Frontier with Application to Lithuanian Dairy Farms

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Outline

- Introduction
- IDF
- Dynamic equation for TFP growth
- Application to Lithuanian dairy farms

Motivation

- DEA allows for non-parametric modelling, whereas SFA allows for stochastic noise.
- Semiparametric modelling allows avoiding assumptions on the functional form yet stochastic noise is accounted for.
- Semiparametric production function can be used in the context of SFA (Fan, Li, 1996; Kumbhakar et al., 2007).
- Li et al. (2002) presented a smooth coefficient (SC) model. Applications by Heshmati et al. (2014), Gong (2018).
- Kumbhakar and Sun (2012) estimated TFP by applying SC model and dynamic equation for the input distance function.
- The error term in KS (2012) has not been decomposed into inefficiency and stochastic noise.

Research Framework

- Two-step estimation procedure:
  - estimating inefficiency in a non-parametric SFA model in level (i.e., static) form, where both the mean and standard deviation of the underlying distributions of inefficiency are functions of the contextual variables,
  - deriving the dynamic (i.e., growth) model of TFP change from the former static counterpart, and estimate the model in the SC setting.
- Such an approach allows for reasonably flexible functional form of the production frontier with clear economic interpretation of the smooth coefficients.
- IDF is estimated.
- We also apply the approach by Kumbhakar and Sun (2013) to estimate the marginal effects of the contextual variables on inefficiency.
- The case of Lithuanian dairy farms is considered.

Technology and Efficiency

- Input requirement set $L(Y) = \{X \in \mathbb{R}^K_+ : X \text{ can produce } Y, \text{ and } Y \in \mathbb{R}^Q_+\}.$
- Input distance function $D(X,Y) = \sup\{\rho : (X/\rho) \in L(Y)\}$
- Imposing homogeneity in X $D \cdot X_1^{-1} = A \cdot F(\tilde{X}, Y, Z, \omega)$
- Imposing homogeneity in Y $D \cdot X_1^{-1} \cdot Y_1 = A \cdot F(\tilde{X}, \tilde{Y}, Z, \omega)$

$$\ln D = \ln A + \ln X_1 - \ln Y_1 + f(\ln \tilde{X}, \ln \tilde{Y}, Z) + \omega,$$

$$-\ln X_{1it} + \ln Y_{1it} = f(\ln \tilde{X}_{it}, \ln \tilde{Y}_{it}, Z_{it}) + \omega_i + v_{it} - u_{it}$$

Estimating the IDF

- The two unknown functions are modelled visa the series regression: $-\ln X_{1it} + \ln Y_{1it} = f(\ln \tilde{X}_{it}, \ln \tilde{Y}_{it}, Z_{it}) + \omega_i - E(u_{it}|Z_{it}) + e_{it},$
- The estimates of the error term are used to obtain the expected inefficiencies:

$$\grave{e}_{it} = E(u_{it}|Z_{it}) + v_{it} - u_{it} \qquad v_{it} \sim iidN(0, \sigma_v^2),\ u_{it} \sim iidN^+(\mu(Z_{it}), \sigma_u^2(Z_{it}))$$

$$\mu(Z_{it}) = c_0 + \delta' Z_{it} \text{ and } \sigma_u(Z_{it}) = \exp(c_1 + \rho' Z_{it})$$

$$E(u_{it}|Z_{it}) = \frac{\mu(Z_{it})a(Z_{it})}{2} + \frac{\sigma_u(Z_{it})a(Z_{it})}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{\mu(Z_{it})}{\sigma_u(Z_{it})}\right)^2\right)$$

$$a(Z_{it}) = [\Phi(\mu(Z_{it})/\sigma_u(Z_{it}))]^{-1}$$

- TE is obtained via the estimator Battese and Coelli (1998), inefficiency change is estimated by following Kumbhakar and Sun (2013).

Dynamic Equation for TFP Growth

- The IDF can be rewritten as

$$u_{it} - \ln X_{1it} + \ln Y_{1it} = f(\ln \tilde{X}_{it}, \ln \tilde{Y}_{it}, \tilde{Z}_{it}, t) + \omega_i + v_{it}.$$

- Differentiating wrt time gives

$$\frac{\partial u_{it}}{\partial t} - \dot{X}_{1it} + \dot{Y}_{1it} = \beta_0(\cdot) + \sum_{k=2}^{K} \beta_k(\cdot)\dot{\tilde{X}}_{kit} + \sum_{q=2}^{Q} \gamma_q(\cdot)\dot{\tilde{Y}}_{qit} + \sum_{p=1}^{P} \varphi_p(\cdot)\nabla_t \tilde{Z}_{pit} + \nu_{it}.$$

- Coefficients can be interpreted as follows

$$\beta_0(\cdot) = \frac{\partial f(\cdot)}{\partial t}; \quad \beta_k(\cdot) = \frac{\partial f(\cdot)}{\partial \ln \tilde{X}_k}; \quad \gamma_q(\cdot) = \frac{\partial f(\cdot)}{\partial \ln \tilde{Y}_q}; \quad \varphi_p(\cdot) = \frac{\partial f(\cdot)}{\partial \tilde{Z}_p}.$$

$$\beta_1(\cdot) = 1 - \textstyle\sum_{k=2}^{K} \beta_k(\cdot) \qquad \gamma_1(\cdot) = -1 - \textstyle\sum_{q=2}^{Q} \gamma_q(\cdot)$$

Data Used

- We use the data from the Farm Accountancy Data Network (FADN) that cover the period of 2004-2016.
- Both specialised milk farms and mixed milk-cattle farms are included in the analysis.
- Two outputs, five inputs, three z-variables plus time trend.

Symbol	Name	Mean	SD	Min	Max
x_1	Labour	5757.49	4902.91	1095.00	93544.00
x_2	Herd size	56.78	68.28	1.84	800.77
x_3	Intermediate consumption	48655.95	78183.26	1752.82	1684415.29
x_4	Assets	72260.01	119547.85	0.41	1494335.84
x_5	Utilised agricultural area	89.29	93.79	2.17	956.77
y_1	Milk output	193.24	287.56	1.50	4867.88
y_2	Other outputs	36951.33	58682.61	98.19	1004395.41
w_1	Labour price	2.05	0.67	0.77	6.90
w_2	Herd price	494.13	424.95	121.37	21438.77
w_3	Intermediate consumption price	1.05	0.16	0.77	1.26
w_4	Assets price	2.75	88.46	0.01	5086.96
w_5	Land price	14.09	12.93	0.03	126.41
p_1	Milk price	229.87	57.09	86.89	520.00
p_2	Other outputs' price	1.00	0.03	0.92	2.41
z_1	HHI	0.67	0.20	0.17	1.00
z_2	Support payments (in log)	9.34	1.02	6.34	12.59
z_3	Farmer's age	47.34	11.21	19.00	99.00
t	Time trend	7.64	3.62	1.00	13.00

1. Total number of observations = 3832.
2. t is defined as year-2003, where year varies from 2004 to 2016.

Decomposing the TFP Change

- Divisia TFP indicator: $\dot{TFP} \equiv \sum_{q=1}^{Q} R_q \dot{Y}_q - \sum_{k=1}^{K} S_k \dot{X}_k$
- By appending and rearranging the dynamic equation, we arrive at

$$\dot{TFP} = \beta_0(\cdot) + \sum_{q=2}^{Q}(R_q + \gamma_q(\cdot))\dot{\tilde{Y}}_q + \sum_{k=2}^{K}(\beta_k(\cdot) - S_k)\dot{\tilde{X}}_k + \sum_{p=1}^{P}\varphi_p(\cdot)\nabla_t \bar{Z}_p - \frac{\partial u}{\partial t} + \nu,$$

- The RHS terms are: TC, scale change (SC), allocative term, external conditions, EC.
- SC can be further broken down into RTS-related term and price mark-up: $\sum_{q=2}^{Q}(R_q + \gamma_q(\cdot))\dot{\tilde{Y}}_q = \sum_{q=1}^{Q}(R_q + \gamma_q(\cdot))\dot{Y}_q$

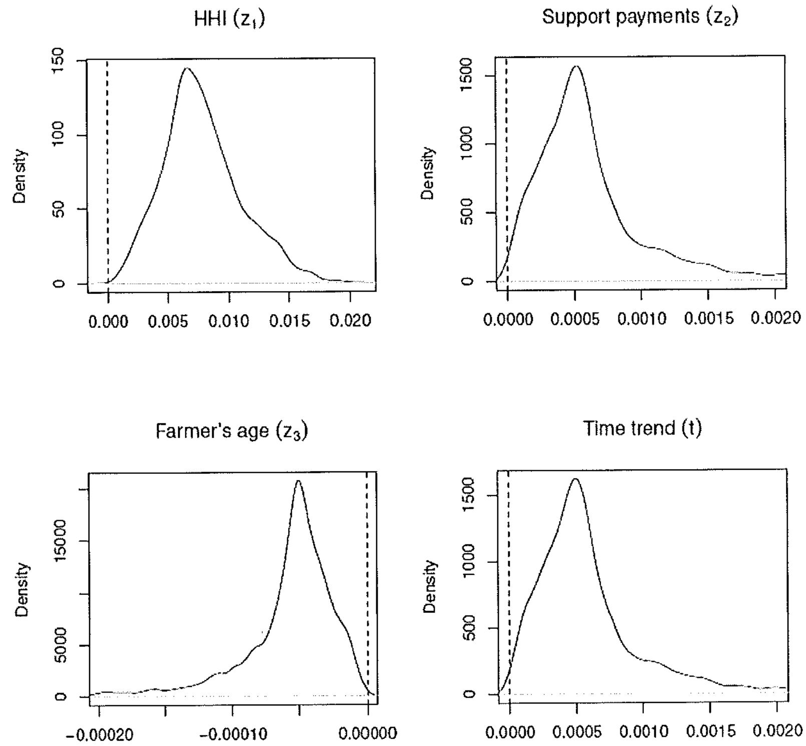
$$= (1 - RTS)\sum_{q=1}^{Q}\gamma_q(\cdot)\dot{Y}_q + \sum_{q=1}^{Q}(R_q + RTS \cdot \gamma_q(\cdot))\dot{Y}_q$$

$$= (1 - RTS)\sum_{q=1}^{Q}\gamma_q(\cdot)\dot{Y}_q + \sum_{q=1}^{Q}\left(R_q - \frac{\gamma_q(\cdot)}{\sum_{q=1}^{Q}\gamma_q(\cdot)}\right)\dot{Y}_q,$$

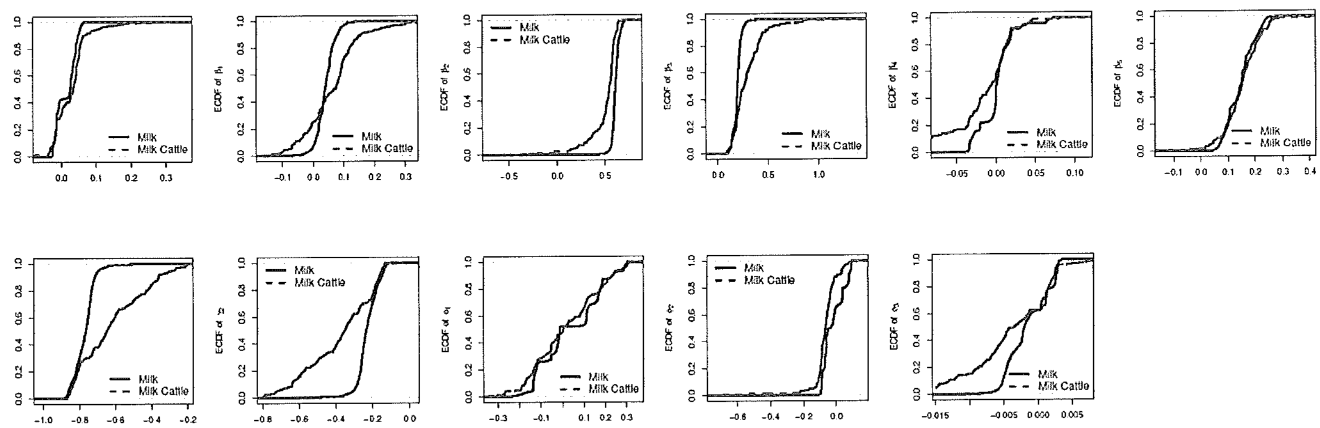
Determinants of the Parameters of the Inefficiency Distribution

Intercept	z_1	z_2	z_3	t
		$\mu(Z)$		
c_0	δ_1	δ_2	δ_3	δ_4
-0.0141	0.0307	-0.0001	-0.0001	-0.0002
(0.0116)	(0.0155)	(0.0019)	(0.0002)	(0.0005)
		$\sigma_u(Z)$		
c_1	ρ_1	ρ_2	ρ_3	ρ_4
-3.9979	0.0053	0.0383	-0.0020	0.0372
(0.3643)	(0.2340)	(0.0406)	(0.0027)	(0.0122)

1. Z variables are described in Section 4.
2. Standard errors are in the parentheses.



ECDF Plots for the SCs

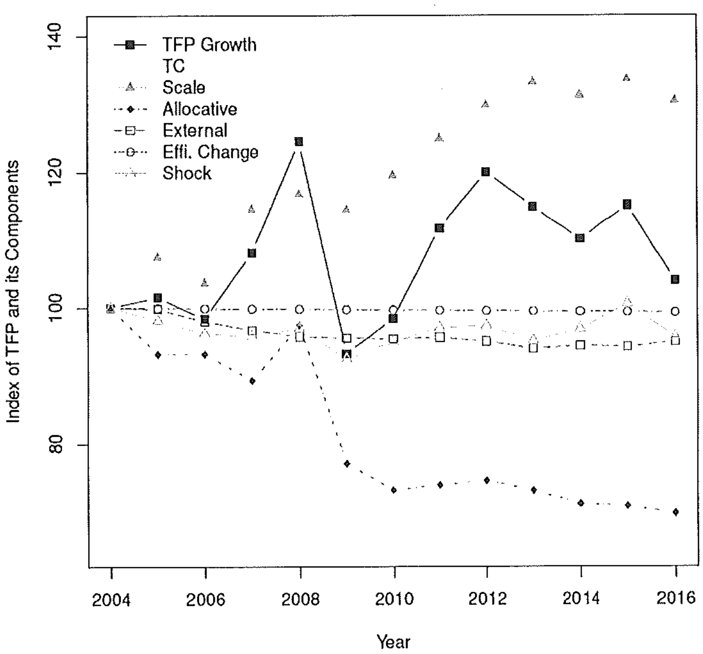


Time Derivatives of the SCs

	$\frac{\partial \beta_0}{\partial t}$	$\frac{\partial \beta_1}{\partial t}$	$\frac{\partial \beta_2}{\partial t}$	$\frac{\partial \beta_3}{\partial t}$	$\frac{\partial \beta_4}{\partial t}$	$\frac{\partial \beta_5}{\partial t}$	$\frac{\partial \gamma_1}{\partial t}$	$\frac{\partial \gamma_2}{\partial t}$	$\frac{\partial \phi_1}{\partial t}$	$\frac{\partial \phi_2}{\partial t}$	$\frac{\partial \phi_3}{\partial t}$
Mean	-0.0082	-0.0061	-0.0015	0.0095	-0.0001	-0.0018	-0.0011	0.0011	0.0259	0.0048	0.0000
	(0.0006)	(0.0007)	(0.0011)	(0.0009)	(0.0005)	(0.0012)	(0.0007)	(0.0007)	(0.0026)	(0.0012)	(0.0001)
Q1	-0.0224	-0.0253	-0.0245	-0.0099	-0.0092	-0.0370	-0.0250	-0.0227	-0.0943	-0.0215	-0.0018
	(0.0005)	(0.0003)	(0.0012)	(0.0003)	(0.0001)	(0.0003)	(0.0008)	(0.0001)	(0.0023)	(0.0005)	(0.0000)
Q2	-0.0024	-0.0084	-0.0060	0.0170	-0.0014	0.0011	0.0050	-0.0050	0.0759	-0.0116	0.0010
	(0.0000)	(0.0004)	(0.0007)	(0.0006)	(0.0002)	(0.0016)	(0.0021)	(0.0022)	(0.0022)	(0.0007)	(0.0001)
Q3	0.0028	0.0061	0.0268	0.0360	0.0169	0.0354	0.0227	0.0250	0.1106	0.0307	0.0015
	(0.0005)	(0.0004)	(0.0006)	(0.0013)	(0.0006)	(0.0009)	(0.0001)	(0.0008)	(0.0022)	(0.0002)	(0.0001)

Bootstrapped standard errors are in the parentheses.

TFP Index and Its Decomposition



Summary Statistics for the TFP Change

	TC	Scale	Allocative	External	Eff. Change	Shock	TFP Growth
Mean	0.0170	0.0176	-0.0200	-0.0024	-0.0006	-0.0005	0.0111
	(0.0008)	(0.0024)	(0.0031)	(0.0006)	(0.0000)	(0.0030)	(0.0047)
Q1	-0.0126	-0.0326	-0.0665	-0.0153	-0.0007	-0.0799	-0.1135
	(0.0004)	(0.0017)	(0.0017)	(0.0004)	(0.0001)	(0.0022)	(0.0031)
Q2	0.0260	0.0049	-0.0027	-0.0022	-0.0005	0.0005	0.0157
	(0.0014)	(0.0009)	(0.0014)	(0.0003)	(0.0000)	(0.0021)	(0.0036)
Q3	0.0420	0.0563	0.0515	0.0109	-0.0003	0.0786	0.1434
	(0.0007)	(0.0014)	(0.0023)	(0.0004)	(0.0000)	(0.0020)	(0.0037)

Bootstrapped standard errors are in the parentheses.

Conclusions

- We considered the semi-parametric analysis of the TFP change.
- We applied the general additive model and SFA to extract the technical inefficiency under the unknown functional form of the IDF.
- We allowed both the mean and standard deviation of the inefficiency distribution to be dependent on the contextual variables.
- The smooth coefficient model was then applied in a dynamic setting to estimate and decompose the TFP change.
- Lithuanian dairy farms maintained TFP growth of 1.1 percent per annum on average during 2004-2016. Much of it was attributed to the TC and scale components.
- The estimation of the smooth coefficient model shed some light on the underlying technology and TFP change. First, that the TC biases towards intermediate consumption indicated the increasing spread of more intensive farming requiring higher level of modernisation.

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